Escape of photons from magnetized cylinders

1975 J. Phys. A: Math. Gen. 8 186

(http://iopscience.iop.org/0305-4470/8/2/008)

J. Phys. A: Math. Gen., Vol. 8, No. 2, 1975. Printed in Great Britain. © 1975

Escape of photons from magnetized cylinders

K D Krori and Jayantimala Barua
Mathematical Physics Forum, Cotton College, Gauhati-1, India

Received 1 July 1974, in final form 19 August 1974

Abstract. We have investigated in this paper the escape of photons from the surface of magnetized cylinders. Here we have considered escape of photons from a radiating solenoid carrying current, and a radiating straight wire carrying current.

1. Introduction

Synge (1966) has shown that for 'gravitationally intense stars' only those photons which are emitted within a slender cylindrical cone can escape to infinity. But in the limit when the surface of the star approaches the Schwarzschild radius only radially moving photons can escape. Banerjee (1968) has found in a corresponding problem involving a cylindrically-symmetric mass distribution that for photons moving in a plane normal to the cylindrical axis there exists a critical value for the mass per unit length of the cylinder below which photons excape to infinity for all angles of emergence. In other cases they are turned back somewhere in their paths, the only exception being those moving radially. Krori and Barua (1974) have recently investigated these two cases considering the spherical and cylindrical distributions of masses to be charged. They have found for charged spherical masses that even when the surface of the sphere approaches the Schwarzschild radius the photons emitted within a slender cylindrical cone may escape to infinity. In the case of charged infinitely long cylinders the mass per unit length does not put any restriction on the photons issuing from the surface in non-radial directions unlike the case of uncharged infinitely long cylinders. But in the limit when the radius approaches a certain value depending on the mass per unit length and the charge per unit length only the radially-moving photons may escape. We have investigated in this paper the escape of photons from magnetized cylinders in a plane perpendicular to the cylindrical axis. For a solenoid carrying current we show that photons may escape only radially. But for a straight wire carrying current it is found that the magnetic field has no effect on the escape of photons.

2. The line elements and the corresponding solutions for magnetized cylinders

2.1. Solenoid carrying current

We take the line element in the form (Safko and Witten 1972)

$$\mathrm{d}s^2 = -(r+\rho)^{2c+2c^2}[1+k_1(r+\rho)^{2+2c}]^2\,\mathrm{e}^{2a}(\mathrm{d}t^2-\mathrm{d}r^2)$$
$$+(r+\rho)^{2c+2}[1+k_1(r+\rho)^{2+2c}]^{-2}\,\mathrm{d}\phi^2+(r+\rho)^{-2c}[1+k_1(r+\rho)^{2+2c}]\,\mathrm{d}z^2 \quad (1)$$

where ρ, c, k_1 and a are constants. c is related to the mass per unit length of the cylinder and k_1 is associated with the magnetic field produced due to current in the solenoid.

The physical component of the magnetic field is given by

$$B_z = \frac{2(1+c)k_1^{1/2}\,\mathrm{e}^a}{(r+\rho)^{c^2}[1+k_1(r+\rho)^{2+2c}]^2}. \tag{2}$$

The current source per unit length of the cylinder is given by

$$I_\phi = 2(1+c)k_1^{1/2}. \tag{3}$$

Now for the null geodesics along which the z coordinate is fixed, we obtain from equation (1), after some calculation, the relation

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\phi}\right)^2 = (r+\rho)^{2(1-c^2)}[1+k_1(r+\rho)^{2+2c}]^{-4}\,\mathrm{e}^{-2a}\left(\frac{M\,\mathrm{e}^{2a}(r+\rho)^{2(1-c^2)}}{[1+k_1(r+\rho)^{2+2c}]^4}-1\right) \tag{4}$$

where M is a constant.

Now if ψ is the angle of inclination made by the light ray with the radial direction, and if we assume that the spatial components of unit vectors along them are $(\mathrm{d}r, \mathrm{d}\theta, 0)$ and $(\mathrm{d}r', 0, 0)$ respectively, then

$$\cos\psi = \frac{g_{11}\,\mathrm{d}r\,\mathrm{d}r'}{(g_{11}\,\mathrm{d}r'^2)^{1/2}(g_{11}\,\mathrm{d}r^2+g_{22}\,\mathrm{d}\phi^2)^{1/2}} = \left[1+\frac{g_{22}}{g_{11}}\left(\frac{\mathrm{d}\phi}{\mathrm{d}r}\right)^2\right]^{-1/2}. \tag{5}$$

In the above we use symbols 1, 2, 3, 4, for r, ϕ, z and t respectively. The equation (5) gives

$$\begin{aligned}\cot^2\psi &= (r+\rho)^{2(c^2-1)}[1+k_1(r+\rho)^{2+2c}]^4\,\mathrm{e}^{2a}\left(\frac{\mathrm{d}r}{\mathrm{d}\phi}\right)^2\\ &= \left(\frac{M\,\mathrm{e}^{2a}(r+\rho)^{2(1-c^2)}}{[1+k_1(r+\rho)^{2+2c}]^4}-1\right).\end{aligned} \tag{6}$$

2.2. *Straight wire carrying current*

Here we take the line element in the form (Safko and Witten 1972)

$$\begin{aligned}\mathrm{d}s^2 = &-(r+\rho)^{2c^2-2c}[k_2+(r+\rho)^{2c}]^2\,\mathrm{e}^{2a}(\mathrm{d}t^2-\mathrm{d}r^2)\\ &+(r+\rho)^{2-2c}[k_2+(r+\rho)^{2c}]^2\,\mathrm{d}\phi^2+(r+\rho)^{2c}[k_2+(r+\rho)^{2c}]^2\,\mathrm{d}z^2\end{aligned} \tag{7}$$

where ρ, c, k_2 and a are constants. Here c is related to the mass per unit length of the wire and k_2 is associated with the magnetic field produced by the current passing through it. The physical component of the magnetic field is given by

$$B_\phi = \frac{2ck_2^{1/2}\,\mathrm{e}^a}{(r+\rho)^{(c+1)^2}[k_2+(r+\rho)^{2c}]^2}. \tag{8}$$

The current source per unit length of the cylinder is given by

$$I_z = 4\pi ck_2^{1/2}. \tag{9}$$

For the null geodesics along which the z coordinate is fixed, we obtain from equation (7), after some calculation, the relation

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\phi}\right)^2 = (r+\rho)^{2(1-c^2)}\,\mathrm{e}^{-2a}[\mathrm{e}^{-2a}N(r+\rho)^{2(1-c^2)}-1], \tag{10}$$

where N is a constant.

Now if ψ is the angle of inclination made by the light ray with the radial direction, then proceeding as in § 2.1 we obtain

$$\cot^2\psi = [\mathrm{e}^{-2a}N(r+\rho)^{2(1-c^2)}-1]. \tag{11}$$

3. Discussions

3.1. Solenoid carrying current

Equation (6) can be written also as

$$\begin{aligned} M &= \mathrm{e}^{-2a}(r+\rho)^{2(c^2-1)}[1+k_1(r+\rho)^{2+2c}]^4\,\mathrm{cosec}^2\,\psi \\ &= \mathrm{e}^{-2a}(r_0+\rho)^{2(c^2-1)}[1+k_1(r_0+\rho)^{2+2c}]^4\,\mathrm{cosec}^2\,\psi_0 \end{aligned} \tag{12}$$

where r_0 and ψ_0 represent the values of r and ψ at the instant of emission from the surface of the cylinder. In view of equation (6) since $(\mathrm{d}r/\mathrm{d}\phi)^2$ is essentially positive M must be larger than $\{\mathrm{e}^{-2a}(r+\rho)^{2(c^2-1)}[1+k_1(r+\rho)^{2+2c}]^4\}$. Hence since, whether c is greater than, equal to or less than 1,

$$\{(r+\rho)^{2(c^2-1)}[1+k_1(r+\rho)^{2+2c}]^4\} \to \infty$$

as $r \to \infty$, it is evident that though the ray may be initially emitted from the surface it must turn back ultimately to the surface. As an only exception, the ray may traverse an infinite distance provided M is infinite which means ψ_0 is equal to zero. Thus we conclude that in the case of a radiating solenoid carrying current, photons issuing in a plane perpendicular to the cylindrical axis may escape to infinity only in radial directions. This means that the restrictions on the photons issuing in a plane perpendicular to the cylindrical axis are more severe when the solenoid has a magnetic field than when it is without the field.

3.2. Straight wire carrying current

Equation (11) can be written as

$$N = \mathrm{e}^{2a}(r+\rho)^{2(c^2-1)}\,\mathrm{cosec}^2\psi = \mathrm{e}^{2a}(r_0+\rho)^{2(c^2-1)}\,\mathrm{cosec}^2\psi_0 \tag{13}$$

where r_0 and ψ_0 represent the values of the r and ψ at the instant of emission from the surface of the wire. Equation (13) does not contain any quantity involving the magnetic field. Hence we conclude that in the case of a radiating straight wire carrying current, photons issuing in a plane perpendicular to the cylindrical axis are unaffected by the magnetic field of the current.

Acknowledgment

The authors are grateful to the Government of Assam, Gauhati, for providing financial assistance at Cotton College, Gauhati, to carry out this piece of work.

References

Banerjee A 1968 *J. Phys.* A: *Gen. Phys.* **1** 495–6
Krori K D and Barua J 1974 *Proc. Ind. Acad. Sci.* A to be published
Safko J L and Witten L 1972 *Phys. Rev.* D **5** 293–300
Synge J L 1966 *Mon. Not. R. Astronom. Soc.* **131** 463–6